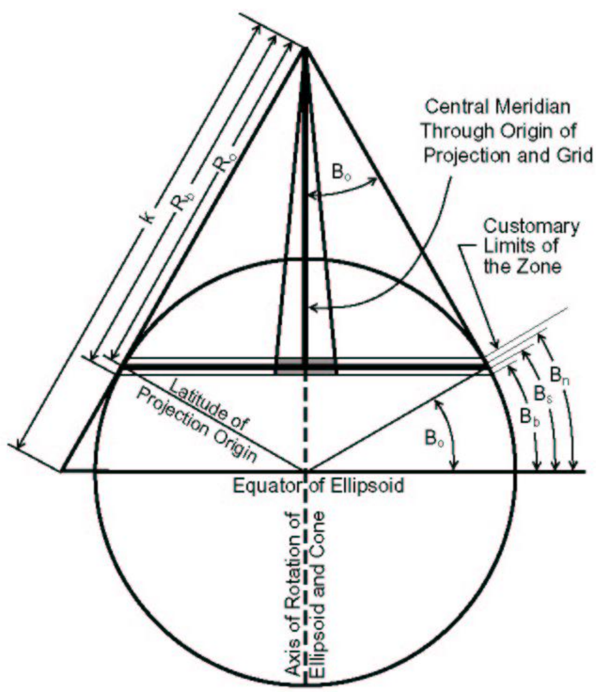
California Land Surveyor's Association

Land Surveyor Review Class
March 28, 2012

State Plane Coordinates



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Video Presentation Outline

The State Plane Coordinate System

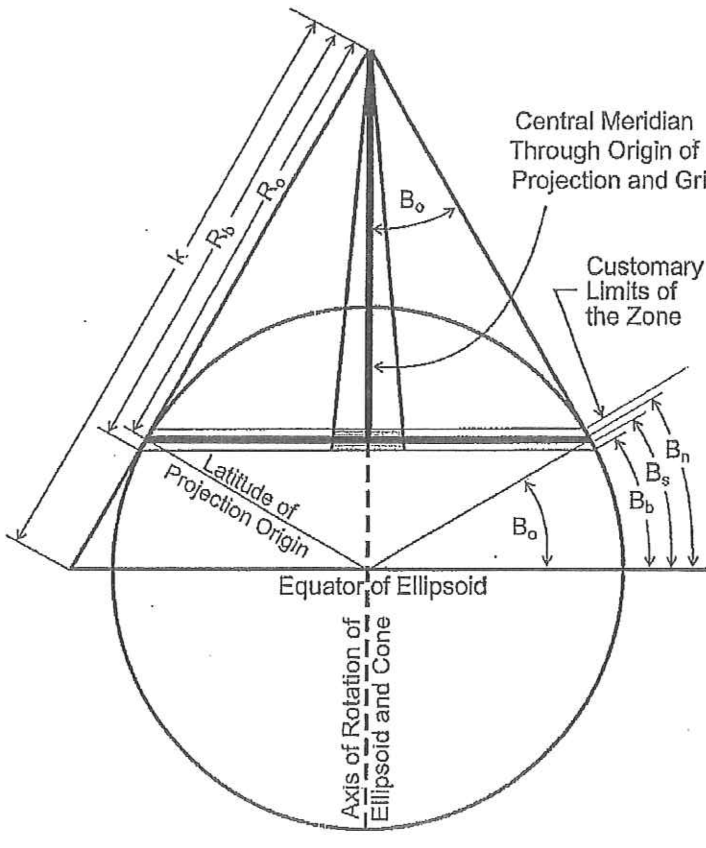


Figure 11-1. Ellipsoid and secant cone.

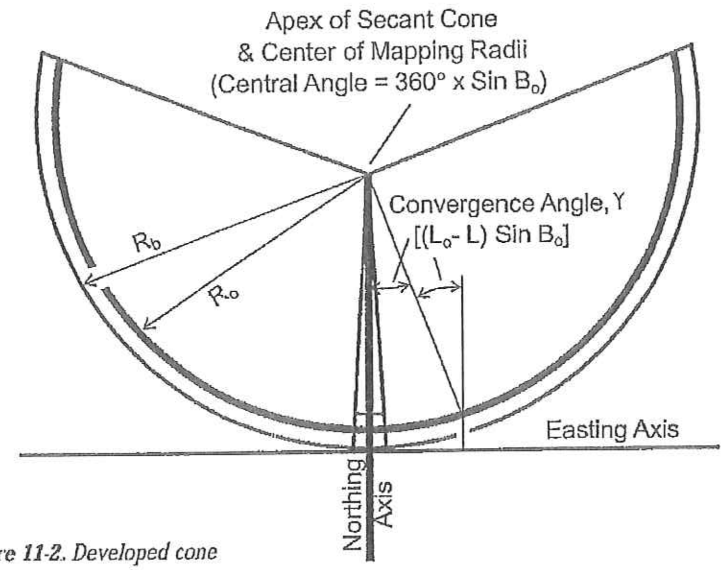


Figure 11-2. Developed cone

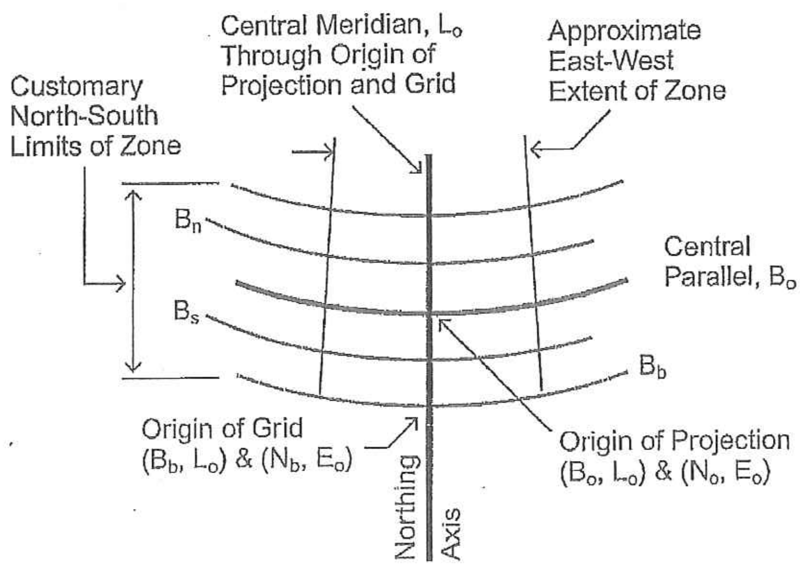


Figure 11-3. Developed cone detail.

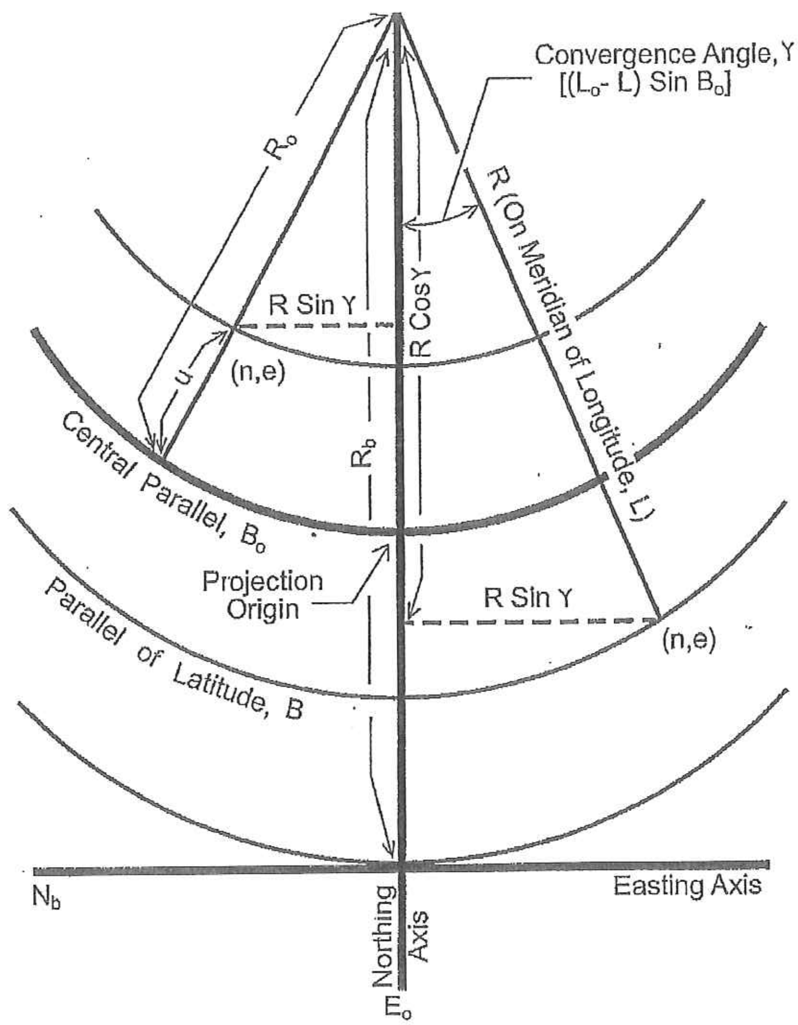


Figure 11-4. Conversion triangle.

Appendix 2

Symbols and Notations

α = geodetic azimuth, the clockwise angle between geodetic north and the geodetic line to the object observed

γ = the plane convergence angle, the major component of the difference between geodetic azimuth and grid azimuth, also sometimes called mapping angle

δ = arc to chord correction, also known as second term or "t - T" (A correction applied to long lines of precise surveys to compensate for distortion of straight lines when projected onto the grid This correction is usually minute and can be neglected for most courses under five miles long)

a = 6378137 m (exact) or 20925604.4742 ft, the equatorial radius of the GRS80 ellipsoid

b = 6356752.314140347 m = 20855444 8840 ft = the semiminor axis of the GRS80 ellipsoid

B = north geodetic latitude of a station, also noted as ø

B_b = north geodetic latitude of the parallel passing through grid origin, a tabled constant

B_n = north geodetic latitude of the northerly standard parallel where the cone intersects the ellipsoid (Line of exact scale)

B_o = the latitude of the central parallel passing through the projection origin, a tabled constant for each zone, also noted as $\__o$

B_s = north geodetic latitude of the southerly standard parallel where the cone intersects the ellipsoid (Line of exact scale)

cf = combined factor for simultaneously applying average ellipsoidal reduction and scale factors

D = ground level, horizontal measured distance

E_o = easting of projection origin and central meridian, 6561666.667 ft or 2000000 0000 m for all zones

e^2 = 0.006694380022903416 = the square of the first eccentricity of the GRS80 ellipsoid

e ft^2 = 0 006739496775481622 = the square of the second eccentricity of the GRS80 ellipsoid

F_1, F_2, F_3 = polynomial coefficients tabled with the zone constants

G_1, G_2, G_3, G_4 = polynomial coefficients for inverse conversion, tabled with the zone constants

h = geodetic height, elevation using the ellipsoid for its datum. Related to MSL datum by the formula, $h = N + H$

H = elevation using the geoid for its datum, this is approximately elevation based on mean sea level

k = point grid scale factor

K = mapping radius on the cone at the equatorial plane of the ellipsoid

k_0 = grid scale factor of the central parallel, B_0, a tabled constant

L = west geodetic longitude of station, also noted as λ

L_c = ellipsoidal chord length

$L_{(grid)}$ = grid length, distance between two points on the grid plane

L_0 = longitude of the central meridian passing through the projection and grid origin, a tabled constant, also noted as λ_0

L_s = measured slope length

L_1, L_2, L_3, L_4 = polynomial coefficients for direct computation, tabled with the zone constants

M_0 = radius of curvature of the ellipsoid in the meridian at the projection origin, scaled to the grid

N = geoid separation or height, the distance at a station from the geoid to the ellipsoid, it is negative within the contiguous 48 states

N_b = northing of grid base, a tabled constant equalling 1640416 667 ft or 500000.0000 m for all zones

N_0 = northing of the projection origin, a tabled constant

p = $1/f$ = flattening[1] = 298.2572221008827 for GRS80

R = mapping radius through a station

R_a = radius of curvature of the ellipsoid in the azimuth

R_b = mapping radius through the grid base, a tabled constant

r_e = ellipsoidal reduction factor, also known as elevation factor

r_0 = geometric mean radius of the ellipsoid at the projection origin, scaled to the grid

R_o = mapping radius through the projection origin, a tabled constant

s = geodetic length, the ellipsoidal arc length of a line

$\sin B_o$ = sine of the latitude of the projection origin, which is also the ratio between γ and longitude in decimals of a degree, a tabled constant

t = grid azimuth, the clockwise angle at a station between the grid meridian (grid north) and the grid line to the observed object (All grid meridians are straight and parallel. Grid azimuths are related to geodetic azimuths by the formula, $t = a - \gamma + \delta$)

T = projected geodetic azimuth (Azimuth of a straight line of the ellipsoid when projected onto the grid is slightly curved.)

u = radial distance on the projection from the station to the central parallel, $R_o - R$

Constants for the Geodetic Reference System of 1980 GRS80 Ellipsoid

a = 6378137 m (exact) = 20925604.4742 ft = the equatorial radius of the ellipsoid

b = 6356752.314140347 m = 20855444.8840 ft = the semiminor axis

p = 1/f = 298.2572221008827 = flattening^{-1}

e^2 = 0.006694380022903416 = the square of the first eccentricity

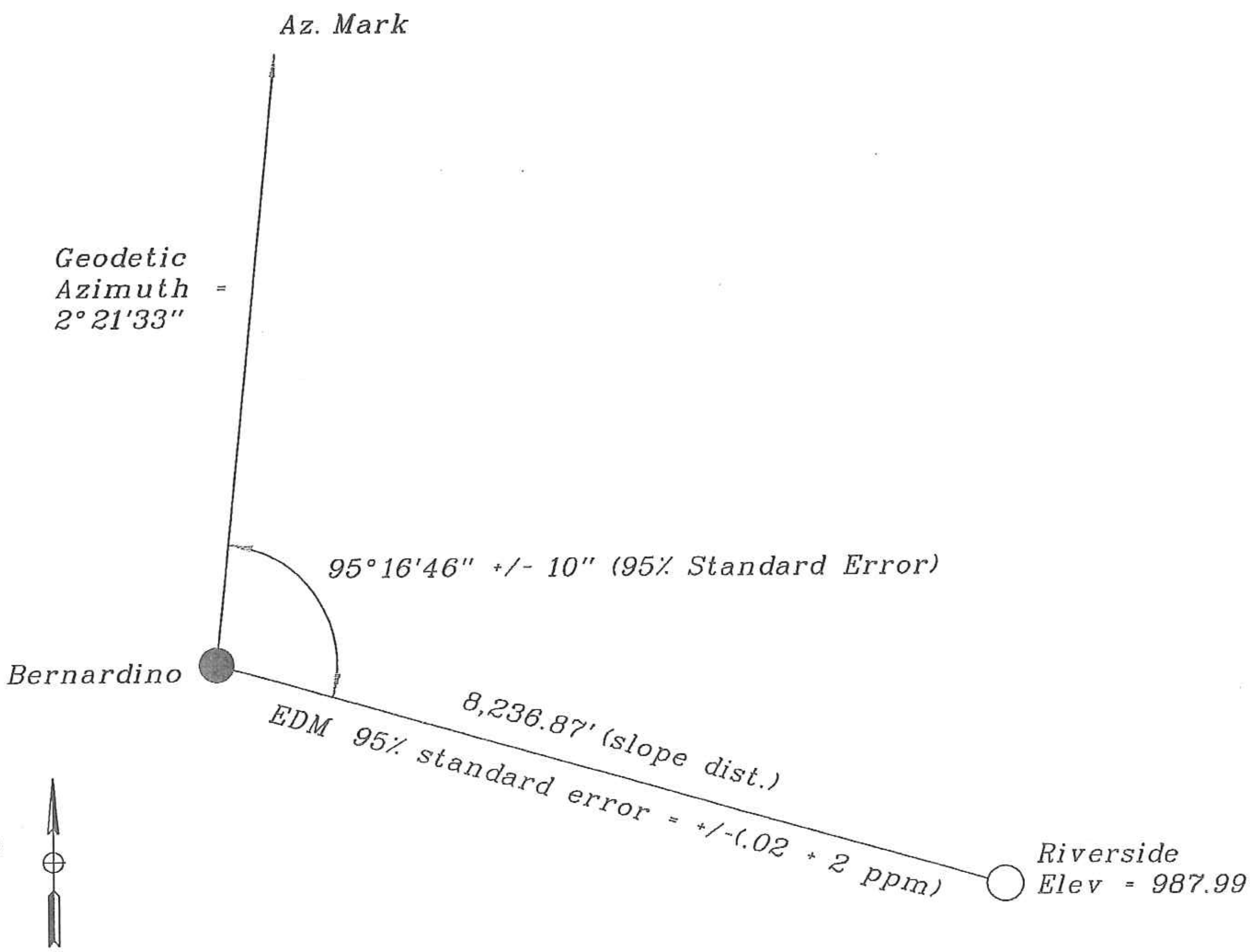
e'^2 = 0.006739496775481622 = the square of the second eccentricity

SAN BERNARDINO / RIVERSIDE CHAPTER CLSA
2008 LS REVIEW WORKSHOP
STATE PLANE COORDINATES

Given:
Station Bernardino
Lat = 34°17'42.54378N
Long = 116°54'17.22556W
Elev = 1253.88'
Geoid Height = -98.29'

Constants for Zone 5:
Central Meridian (Lo) = 118°00'
Rb = 30648744.932'
Eo = 6561666.667'
SinBo = 0.570011896174
Nb = 1640416.667'
ro = 20899279.068
ko = 0.999922127209

Required:
Determine the coordinates of Station Riverside in CCS83, Zone 5, and the 95% standard error of those coordinates. Express the results in the form N = y,yyy,yyy.yy' +/- y.yy' and E = x,xxx,xxx.xx' +/- x.xx'. Assume the coordinates of Station Bernardino are fixed.



bernardino-riverside.dgn 1/28/2008 10:58:39 AM

ZONE 5 CALIFORNIA MAPPING RADIUS
North American Datum 1983

K = 13282624.8345   L = .570011896174   e = .0818191910428

34 DEGREES

MIN	R (meter)	TAB DIFF (per sec)	R (US surv ft)	TAB DIFF (per sec)	SCALE FACTOR
		30.81202		101.08909	
0	9,286,292.9514		30,466,779.46		1.00000739
		30.81199		101.08899	
1	9,284,444.2322		30,460,714.12		1.00000365
		30.81196		101.08889	
2	9,282,595.5149		30,454,648.78		1.00000000
		30.81193		101.08880	
3	9,280,746.7992		30,448,583.46		.99999643
		30.81190		101.08872	
4	9,278,898.0851		30,442,518.13		.99999294
		30.81188		101.08865	
5	9,277,049.3722		30,436,452.82		.99998954
		30.81186		101.08858	
6	9,275,200.6606		30,430,387.50		.99998622
		30.81184		101.08853	
7	9,273,351.9500		30,424,322.19		.99998299
		30.81183		101.08847	
8	9,271,503.2403		30,418,256.88		.99997983
		30.81182		101.08844	
9	9,269,654.5313		30,412,191.57		.99997677
		30.81181		101.08840	
10	9,267,805.8229		30,406,126.27		.99997378
		30.81180		101.08839	
11	9,265,957.1148		30,400,060.97		.99997088
		30.81180		101.08837	
12	9,264,108.4071		30,393,995.67		.99996806
		30.81179		101.08836	
13	9,262,259.6995		30,387,930.36		.99996533
		30.81180		101.08837	
14	9,260,410.9917		30,381,865.06		.99996268
		30.81180		101.08838	
15	9,258,562.2838		30,375,799.76		.99996011
		30.81180		101.08840	
16	9,256,713.5755		30,369,734.46		.99995763
		30.81181		101.08843	
17	9,254,864.8666		30,363,669.15		.99995523
		30.81182		101.08846	
18	9,253,016.1572		30,357,603.84		.99995292
		30.81184		101.08851	
19	9,251,167.4468		30,351,538.53		.99995068

Survey/Mapping Sciences 221
State Plane Coordinates
Formulas for Projection Table Computations

Geodetic Coordinates to State Plane Coordinates:

Determine mapping radius (R):

$R = R_{TABLED} - (\text{Seconds of Latitude of Station} \times \text{Tab Diff})$

R = mapping radius of station
R_{TABLED} = mapping radius of station, to the even minute of latitude, determined from Projection Tables
Tabular Difference (Tab Diff) = linear distance of 1" of latitude (at latitude of station) from Projection Tables

Determine plane convergence (γ):

$\gamma = (L_o - L) \sin B_o$ (carry all significant digits)

γ = convergence angle
L_o = longitude of central meridian, longitude of projection and grid origin, a tabled constant.
L = west longitude of station (λ)
$\sin B_o$ = sine of the latitude of the projection origin, a tabled constant

Determine the northing and easting:

$n = R_b + N_b - R \cos\gamma$
$e = E_o + R \sin\gamma$

n = northing of station
R_b = mapping radius of the grid base, a tabled constant
N_b = northing of the grid base, a tabled constant
R = mapping radius of the station
γ = convergence angle
e = easting of station
E_o = easting of the projection and grid origin, a tabled constant

State Plane Coordinates to Geodetic Coordinates:

Determine plane convergence (γ):

$\gamma = \arctan[(e - E_o) \div (R_b - n + N_b)]$ (carry all significant digits)

γ = convergence angle at station
e = easting of station
E_o = easting of projection and grid origin, a tabled constant
R_b = mapping radius of the grid base, a tabled constant
n = northing of station
N_b = northing of grid base, a tabled constant

Determine longitude (L) (λ):

$L = L_o - (\gamma \div \sin B_o)$

L = west longitude of station (λ)
L_o = longitude of central meridian, longitude of projection and grid origin, a tabled constant
γ = convergence angle at station
$\sin B_o$ = sine of the latitude of the projection origin, a tabled constant.

Determine latitude, (B) (φ)

$B = B_{TABLED} + [(R_{TABLED} - R) \div \text{tab diff}]$

B = north latitude of station (φ)
R = mapping radius of station
R_{TABLED} = mapping radius of station, to the even minute of latitude, determined from Projection Tables
B_{TABLED} = latitude of station, to the even minute, determined from Projection Tables
Tabular Difference (Tab Diff) = linear distance of 1" of latitude (at latitude of station) from Projection Tables

Reducing Measured Distance to Geodetic Length to Grid Length

Determine radius of curvature of the ellipsoid (R_α)

$R_\alpha(\pm) = r_o \div k_o$

R_α = geometric mean radius of curvature of the ellipsoid at the projection origin
r_o = geometric mean radius of the ellipsoid at the projection origin, scaled to grid
k_o = grid scale factor of the central parallel

Determine ellipsoidal reduction factor (r_e), (elevation factor, EF)

$r_e = R_\propto \div (R_\propto + N + H)$

r_e = ellipsoidal reduction factor
R_α = geometric mean radius of curvature of the ellipsoid at the projection origin
N = geoid separation
H = elevation, based on mean sea level, to which measured line was reduced to horizontal

Determine ellipsoidal chord length (L_c)

$L_c = r_e \times D$

L_c = ellipsoidal chord length
r_e = ellipsoidal reduction factor
D = ground level, horizontal measured distance

or

$L_c = R_\propto [(L_s^2 - \Delta h^2) \div (R_\propto + h_1)(R_\propto + h_2)]^{1/2}$

L_c = ellipsoidal chord length
R_α = geometric mean radius of curvature of the ellipsoid at the projection origin
L_s = measure slope distance
$\Delta h = h_1 - h_2$ = difference of geodetic heights
h_1, h_2 = geodetic height of station and foresight, where h = N + H

Determine correction of ellipsoidal chord length to geodetic length

$c = L_c^3 \div 24R_\propto^2$

c = correction, in same units as used for L_c and $R_\propto$
L_c = ellipsoidal chord length
R_α = geometric mean radius of curvature of the ellipsoid at the projection origin

Determine geodetic length (s)

$s = L_c + c$

L_c = ellipsoidal chord length
c = correction, in same units as used for L_c and R_α

Determine point scale factor (k)

Interpolated from latitude of the station using the projection tables

Determine grid length ($L_{(grid)}$)

$L_{(grid)} = s \times k$

$L_{(grid)}$ = length on grid
s = geodetic distance
k = point scale factor of midpoint of line

Determining Grid Azimuth

Determine the plane convergence (γ)

$\gamma = \arctan[(e - E_o) \div (R_b - n + N_b)]$ (if plane coordinates are known)

or

$\gamma = \sin B_o(L_o - L)$ (if longitude is known)

Determine the second term (δ)

$\delta_{1\,2} = \{\sin[(2B_1 + B_2) \div 3] - \sin B_o\} (L_1 - L_2) \div 2$

$\delta_{1\,2}$ = second term azimuth correction, in decimal degrees, from station to foresight
B_1, B_2 = geodetic latitude of station and foresight, respectively
L_1, L_2 = geodetic longitude of station and foresight, respectively

or

$\delta_{1\,2} = [n_1 - N_o + (n_2 - n_1) \div 3] (e_2 - e_1)(6.546 \times 10^{-14})$ (if in feet)
$\delta_{1\,2} = [n_1 - N_o + (n_2 - n_1) \div 3] (e_2 - e_1)(7.046 \times 10^{-13})$ (if in meters)

$\delta_{1\,2}$ = second term azimuth correction, in decimal degrees, from station to foresight
n_1, n_2 = northing of station and foresight, respectively
e_1, e_2 = easting of station and foresight, respectively

Determine grid azimuth (t)

$t = \alpha - \gamma + \delta$

t = grid azimuth
α = geodetic azimuth
γ = plane convergence
δ = second term correction

Survey/Mapping Sciences 221
State Plane Coordinates
Formulas for Polynomial Coefficients Computations

Geodetic Coordinates to State Plane Coordinates:

Determine radial difference (u)

$\Delta B = B - B_o$ (in decimal degrees)
$u = L_1\Delta B + L_2\Delta B^2 + L_3\Delta B^3 + L_4\Delta B^4$

B = north latitude of station (φ)
B_o = latitude of the projection origin, the central parallel, a tabled constant
u = radial distance from station to the central parallel, $R_o - R$
L_1, L_2, L_3, L_4 = polynomial coefficients for direct computation, tabled with the zone constants.

Determine mapping radius (R):

$R = R_o - u$

R = mapping radius of station
R_o = mapping radius of the projection origin
u = radial distance from station to central parallel, $R_o - R$

Determine plane convergence (γ):

$\gamma = (L_o - L)\sin B_o$ (carry all significant digits)

γ = convergence angle
L_o = longitude of central meridian, longitude of projection and grid origin, a tabled constant.
L = west longitude of station (λ)
$\sin B_o$ = sine of the latitude of the projection origin, a tabled constant

Determine the northing and easting:

$n = R_b + N_b - R\cos\gamma$
$e = E_o + R\sin\gamma$

n = northing of station
R_b = mapping radius of the grid base, a tabled constant
N_b = northing of the grid base, a tabled constant
R = mapping radius of the station
γ = convergence angle
e = easting of station
E_o = easting of the projection and grid origin, a tabled constant

State Plane Coordinates to Geodetic Coordinates:

Determine plane convergence (γ):

$\gamma = \arctan[(e - E_o) \div (R_b - n + N_b)]$ (carry all significant digits)

γ = convergence angle at station
e = easting of station
E_o = easting of projection and grid origin, a tabled constant
R_b = mapping radius of the grid base, a tabled constant
n = northing of station
N_b = northing of grid base, a tabled constant

Determine longitude (L) (λ):

$L = L_o - (\gamma \div \sin B_o)$

L = west longitude of station (λ)
L_o = longitude of central meridian, longitude of projection and grid origin, a tabled constant
γ = convergence angle at station
$\sin B_o$ = sine of the latitude of the projection origin, a tabled constant.

Determine radial difference (u):

$u = n - N_o - [(e - E_o) \tan(\gamma \div 2)]$

u = radial distance from station to the central parallel, (R_o - R)
n = northing of station
N_o = northing of the projection origin, a tabled constant.
e = easting of station
E_o = easting of projection and grid origin, a tabled constant
γ = convergence angle at station

Determine latitude, (B) (φ)

$B = B_o + G_1u + G_2u^2 + G_3u^3 + G_4u^4$

B = north latitude of station (φ)
B_o = latitude of the projection origin, the central parallel, a tabled constant
u = radial distance from station to the central parallel, $R_o - R$
G_1, G_2, G_3, G_4 = polynomial coefficients for inverse computation, tabled with the zone constants.

Reducing Measured Distance to Geodetic Length to Grid Length

Determine radius of curvature of the ellipsoid (R_α)

$R_\alpha(\pm) = r_o \div k_o$

R_α = geometric mean radius of curvature of the ellipsoid at the projection origin
r_o = geometric mean radius of the ellipsoid at the projection origin, scaled to grid
k_o = grid scale factor of the central parallel

Determine ellipsoidal reduction factor (r_e), (elevation factor, EF)

$r_e = R_\propto \div (R_\propto + N + H)$

r_e = ellipsoidal reduction factor
R_α = geometric mean radius of curvature of the ellipsoid at the projection origin
N = geoid separation
H = elevation, based on mean sea level, to which measured line was reduced to horizontal

Determine ellipsoidal chord length (L_c)

$L_c = r_e \times D$

L_c = ellipsoidal chord length
r_e = ellipsoidal reduction factor
D = ground level, horizontal measured distance

or

$L_c = R_\propto [(L_s^2 - \Delta h^2) \div (R_\propto + h_1)(R_\propto + h_2)]^{1/2}$

L_c = ellipsoidal chord length
R_α = geometric mean radius of curvature of the ellipsoid at the projection origin
L_s = measure slope distance
$\Delta h = h_1 - h_2$ = difference of geodetic heights
h_1, h_2 = geodetic height of station and foresight, where $h = N + H$

Determine correction of ellipsoidal chord length to geodetic length

$c = L_c^3 \div 24R_\propto^2$

c = correction, in same units as used for L_c and $R_\propto$
L_c = ellipsoidal chord length
R_α = geometric mean radius of curvature of the ellipsoid at the projection origin

Determine geodetic length (s)

$s = L_c + c$

L_c = ellipsoidal chord length
c = correction, in same units as used for L_c and R_∞

Determine point scale factor (k)

$u = L_1\Delta B + L_2\Delta B^2 + L_3\Delta B^3 + L_4\Delta B^4$ (Geodetic Lat. and Long. are known)
or
$u = n - N_o - [(e - E_o)\tan(\gamma \div 2)]$ (γ and plane coordinates are known)
or
$u = R_o - [(R_b + N_b - n)^2 + (e - E_o)^2]^{1/2}$ (plane coordinates are known)

u = radial distance from station to the central parallel, $R_o - R$

$k = F_1 + F_2u^2 + F_3u^3$

k = point scale factor
F_1, F_2, F_3 = polynomial coefficients tabled with the zone constants
u = radial distance from station to the central parallel, $R_o - R$

Determine grid length ($L_{(grid)}$)

$L_{(grid)} = s \times k$

$L_{(grid)}$ = length on grid
s = geodetic distance
k = point scale factor of midpoint of line

Determining Grid Azimuth

Determine the plane convergence (γ)

$\gamma = \arctan[(e - E_o) \div (R_b - n + N_b)]$ (if plane coordinates are known)

or

$\gamma = \sin B_o(L_o - L)$ (if longitude is known)

Determine the second term (δ)

$\delta_{1\,2} = \{\sin[(2B_1 + B_2) \div 3] - \sin B_o\} (L_1 - L_2) \div 2$

$\delta_{1\,2}$ = second term azimuth correction, in decimal degrees, from station to foresight
B_1, B_2 = geodetic latitude of station and foresight, respectively
L_1, L_2 = geodetic longitude of station and foresight, respectively

or

$\delta_{1\,2} = [n_1 - N_o + (n_2 - n_1) \div 3] (e_2 - e_1)(6.546 \times 10^{-14})$ (if in feet)
$\delta_{1\,2} = [n_1 - N_o + (n_2 - n_1) \div 3] (e_2 - e_1)(7.046 \times 10^{-13})$ (if in meters)

$\delta_{1\,2}$ = second term azimuth correction, in decimal degrees, from station to foresight
n_1, n_2 = northing of station and foresight, respectively
e_1, e_2 = easting of station and foresight, respectively

Determine grid azimuth (t)

$t = \alpha - \gamma + \delta$

t = grid azimuth
α = geodetic azimuth
γ = plane convergence
δ = second term correction

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 5 CA05 ZONE# 0405

Meters			US Survey Feet		
B_s	=	34° 02' N	B_s	=	34° 02' N
B_n	=	35° 28' N	B_n	=	35° 28' N
B_b	=	33° 30' N	B_b	=	33° 30' N
L_o	=	118° 00' W	L_o	=	118° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	34.7510553142° N	B_o	=	34.7510553142° N
$SinB_o$	=	0.570011896174	$SinB_o$	=	0.570011896174
R_b	=	9341756.1389 m	R_b	=	30648744.932 ft
R_o	=	9202983.1099 m	R_o	=	30193453.753 ft
N_o	=	638773.0290 m	N_o	=	2095707.846 ft
K	=	13282624.8345 m	K	=	43578078.311 ft
k_o	=	0.999922127209	k_o	=	0.999922127209
M_o	=	6355670.9697 m	M_o	=	20851897.173 ft
r_o	=	6370113. m	r_o	=	20899279.068 ft
L_1	=	110927.3840	L_1	=	363934.2590
L_2	=	9.12439	L_2	=	29.9356
L_3	=	5.64805	L_3	=	18.5303
L_4	=	0.017445	L_4	=	0.057234
G_1	=	9.014906468E-06	G_1	=	2.747748987E-06
G_2	=	-6.68534E-15	G_2	=	-6.21091E-16
G_3	=	-3.72796E-20	G_3	=	-1.05565E-21
G_4	=	-8.6394E-28	G_4	=	-7.4567E-30
F_1	=	0.999922127209	F_1	=	0.999922127209
F_2	=	1.23221E-14	F_2	=	1.14477E-15
F_3	=	4.41E-22	F_3	=	1.25E-23

The customary limits of the zone are from 33° 30' N to 36° 20' N.